

Stochastic emergence of inflaton fluctuations in a SdS primordial universe with large-scale repulsive gravity from a 5D vacuum.

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Abstract

We develop a stochastic approach to study scalar field fluctuations of the inflaton field in an early inflationary universe with a black-hole (BH), which is described by an effective 4D Schwarzschild-de Sitter (SdS) metric. This effective 4D metric is the induced metric on a 4D hypersurface, here representing our universe, which is obtained from a 5D Ricci-flat SdS static metric, after implementing a planar coordinate transformation. On this background we found that at the end of inflation, the squared fluctuations of the inflaton field are not exactly scale independent and result sensitive to the mass of the BH.

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I. INTRODUCTION

In stochastic inflation the dynamics of the inflaton field is described by a second order stochastic equation, where the emergence of a long-wave classical field that drives inflation is subject to a short-wave classical noise. Starobinsky [1] has noted that under certain assumptions, the splitting of the scalar field into long-wavelength and short-wavelength components leads to a quantum Langevin equation that could become classical stochastic dynamics for the long-wavelength modes of the scalar field. This approach emphasizes the role of the quantum fluctuations as the driving forces of the inflation. It considers as a main ingredient the set of long-wavelength modes as a whole, from which the coarse-grained field emerges. This coarse-grained field is assumed to have a highly classical behavior, but the inflow of short-wavelength modes alters its evolution in a random way. Furthermore, the quantum fluctuations of the short-wavelength field, give place to cosmological density perturbations that could be the origin of the structure of the universe [2]. The coarse-graining representation of the inflaton field has played an important role in 4D standard inflationary cosmology [3], in 5D inflationary cosmology from modern Kaluza-Klein theory [4], and in extensions to vectorial fields more recently implemented in the framework of Gravitoelectromagnetic Inflation [5].

On the other hand, in the last years theories with extra dimensions have become quite popular in the scientific community [6]. In particular, some brane scenarios [7], and the induced matter (IM) theory of gravity [8], have been subject of a great amount of research. Even when both theories have different physical motivations for the introduction of a large extra dimension, they are equivalent each other, and predict identical non-local and local high energy corrections to general relativity in 4D, and usual matter in 4D is a consequence of the metric dependence on the fifth extra coordinate [9].

On the basis of the IM theory, we have recently shown in [10] that there exists a 5D SdS BH solution of the theory, from which we can derive a 4D cosmological model where gravity manifests itself as attractive on small (planetary and astrophysical) scales, but repulsive on very large (cosmological) scales. This behavior of gravity derived from this 5D framework, leave us to put on the desk the following question: can repulsive gravity be considered as a strong candidate for explaining the large-scale accelerated expansion of the universe in the past and today? To answer this question let us to start by defining the physical vacuum via

the 5D Ricci-flat metric [11]:

$$dS_5^2 = \left(\frac{\psi}{\psi_0}\right)^2 \left[c^2 f(R) dT^2 - \frac{dR^2}{f(R)} - R^2 (d\theta^2 + \sin^2 \theta d\phi^2) \right] - d\psi^2. \quad (1)$$

Here, $f(R) = 1 - (2G\zeta\psi_0/Rc^2) - (R/\psi_0)^2$ is a dimensionless function, $\{T, R, \theta, \phi\}$ are the usual local spacetime spherical coordinates employed in general relativity and ψ is the non-compact space-like extra dimension. In this line element ψ_0 is an arbitrary constant with length units, c denotes the speed of light, and the constant parameter ζ has units of $(mass)(length)^{-1}$. This static metric is a 5D extension of the 4D SdS metric. In order to get this metric written on a dynamical chart coordinate $\{t, r, \theta, \phi\}$, we use the coordinate transformation given by [12]

$$R = ar \left[1 + \frac{G\zeta\psi_0}{2ar} \right]^2, \quad T = t + H \int^r dR \frac{R}{f(R)} \left(1 - \frac{2G\zeta\psi_0}{R} \right)^{-1/2}, \quad \psi = \psi, \quad (2)$$

$a(t) = e^{Ht}$ being the scale factor, and H the Hubble constant. Thus the line element (1) can be written in terms of the conformal time τ as

$$dS_5^2 = \left(\frac{\psi}{\psi_0}\right)^2 \left[F(\tau, r) d\tau^2 - J(\tau, r) (dr^2 + r^2 (d\theta^2 + \sin^2 \theta d\phi^2)) \right] - d\psi^2, \quad (3)$$

where the metric functions $F(\tau, r)$ and $J(\tau, r)$ are given by

$$F(\tau, r) = a^2(\tau) \left[1 - \frac{G\zeta\psi_0}{2a(\tau)r} \right]^2 \left[1 + \frac{G\zeta\psi_0}{2a(\tau)r} \right]^{-2}, \quad J(\tau, r) = a^2(\tau) \left[1 + \frac{G\zeta\psi_0}{2a(\tau)r} \right]^4, \quad (4)$$

with $d\tau = a^{-1}(\tau)dt$ and $a(\tau) = -1/(H\tau)$, so that the Hubble parameter is a constant given by $H = a^{-2} \frac{da}{d\tau}$. As it was shown in [11], for certain values of ζ and ψ_0 , both metrics (1) and (3) have two natural horizons. The inner horizon is the analogous of the Schwarzschild horizon and the external one is the analogous of the Hubble horizon.

Now we consider a 5D massless scalar field which is free of any interactions: ${}^{(5)}\square\varphi = 0$. We assume that $\varphi(\tau, r, \theta, \phi, \psi)$ can be separated in the form $\varphi(\tau, r, \theta, \phi, \psi) \sim \Phi(\tau, r)G(\theta, \phi)\Omega(\psi)$, so that the expression ${}^{(5)}\square\varphi = 0$ leaves to

$$\left(\frac{\psi}{\psi_0}\right)^{-2} \frac{d}{d\psi} \left[\left(\frac{\psi}{\psi_0}\right)^4 \frac{d\Omega}{d\psi} \right] + M^2 \Omega = 0, \quad (5)$$

$$\begin{aligned} & \frac{1}{\sqrt{FJ}} \frac{\partial}{\partial \tau} \left(\sqrt{\frac{J^3}{F}} \frac{\partial \Phi}{\partial \tau} \right) - \frac{1}{2} \left(\frac{1}{F} \frac{\partial F}{\partial r} + \frac{1}{J} \frac{\partial J}{\partial r} \right) \frac{\partial \Phi}{\partial r} - \frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial \Phi}{\partial r} \right) \\ & - \left(\frac{l(l+1)}{r^2} - M^2 J \right) \Phi = 0, \end{aligned} \quad (6)$$

where $M^2 > 0$ is a separation constant with mass units and l is an integer dimensionless parameter related with the angular momentum.

II. THE DYNAMICS OF φ ON THE 4D HYPERSURFACE Σ

Assuming that the 5D spacetime can be foliated by a family of hypersurfaces $\Sigma : \psi = \psi_0$, from the metric (3) we obtained that the 4D induced metric on every leaf Σ is given by

$$dS_4^2 = F(\tau, r)d\tau^2 - J(\tau, r)[dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2)], \quad (7)$$

where the metric functions $F(\tau, r)$ and $J(\tau, r)$ can be now written in terms of the physical mass $m = \zeta\psi_0$ (introduced by the first time in [11]), in the form

$$F(\tau, r) = a^2(\tau) \left[1 - \frac{Gm}{2a(\tau)r}\right]^2 \left[1 + \frac{Gm}{2a(\tau)r}\right]^{-2}, \quad J(\tau, r) = a^2(\tau) \left[1 + \frac{Gm}{2a(\tau)r}\right]^4. \quad (8)$$

The induced metric (7) has a Ricci scalar ${}^{(4)}\mathcal{R} = 12H^2$. describes a black hole in an expanding universe, where the expansion is driven by a kind of cosmological constant, whose value in general depends of the value of ψ_0 . According to [11], this metric also indicates that there exists a length scale that separates regions on which gravity changes from attractive to repulsive. This length scale is called the gravitational-antigravitational radius, which in the coordinates (T, R) is given by $R_{ga} = (Gm\psi_0^2)^{1/3}$. With the help of (2), in the new coordinates (τ, r) this radius must obey the relation

$$r_{ga} = \frac{1}{2a(\tau)} \left[R_{ga} - Gm \pm \sqrt{R_{ga}^2 - 2GmR_{ga}} \right], \quad (9)$$

where r_{ga} is denoting the gravitational-antigravitational radius in the new coordinates and the solution with the minus sign is not physical. In order to r_{ga} to be a real value quantity, we require the condition $R_{ga}^2 - 2GmR_{ga} \geq 0$ to be hold. This condition can be rewritten in the form $m^2 \leq \frac{\psi_0^2}{8G^2}$. Thus, if we consider the foliation $\psi_0 = c^2/H$ and the fact that for $c = \hbar = 1$ the Newtonian constant is $G = M_p^{-2}$, this condition leaves now to the restriction $\epsilon = \frac{mH}{M_p^2} \leq \frac{1}{2\sqrt{2}} \simeq 0.353553$. This way, the model allows to consider objects whose mass satisfies the parameter condition: $\epsilon = GmH \ll 1$. For these values, R_{ga} is smaller than the size of our universe horizon. The same restriction has been used in [13] with different motivation.

Now, from (5) and (6), the 4D induced field equation reads

$$\frac{1}{\sqrt{FJ^3}} \frac{\partial}{\partial \tau} \left[\sqrt{\frac{J^3}{F}} \frac{\partial \bar{\varphi}}{\partial \tau} \right] - \frac{1}{2} \left(\frac{1}{FJ} \frac{\partial F}{\partial r} + \frac{1}{J^2} \frac{\partial J}{\partial r} \right) \frac{\partial \bar{\varphi}}{\partial r} - \frac{1}{J} \nabla^2 \bar{\varphi} + M^2 \bar{\varphi} = 0, \quad (10)$$

¹ $M_p = 1.2 \times 10^{19}$ GeV is the Planckian mass.

where $\bar{\varphi}(\tau, r, \theta, \phi) = \varphi(\tau, r, \theta, \phi, \psi_0)$ is the effective scalar field induced on the generic hypersurface Σ , which we shall identify with the inflaton field. It can be easily seen from (10) that M here corresponds to the physical mass of $\bar{\varphi}$. We can expand the field $\bar{\varphi}$ as

$$\bar{\varphi}(\vec{r}, \tau) = \int_0^\infty dk \sum_{lm} \left[a_{klm} \bar{\Phi}_{klm}(\vec{r}, \tau) + a_{klm}^\dagger \bar{\Phi}_{klm}^*(\vec{r}, \tau) \right], \quad (11)$$

where $\bar{\Phi}_{klm}(\vec{r}, \tau) = k^2 j_l(kr) \bar{\Phi}_{kl}(\tau) Y_{lm}(\theta, \phi)$, $Y_{lm}(\theta, \phi)$ are the spherical harmonics, $j_l(kr)$ are the spherical Bessel functions and the annihilation and creation operators obey: $[a_{klm}, a_{k'l'm'}^\dagger] = \delta(k - k') \delta_{ll'} \delta_{mm'}$, $[a_{klm}, a_{k'l'm'}] = [a_{klm}^\dagger, a_{k'l'm'}^\dagger] = 0$. Hence, using the addition theorem for spherical harmonics, we obtain for the mean squared fluctuations

$$\langle 0 | \bar{\varphi}^2(\vec{r}, \tau) | 0 \rangle = \int_0^\infty \frac{dk}{k} \sum_l \frac{2l+1}{4\pi} k^5 j_l^2(kr) |\bar{\Phi}_{kl}(\tau)|^2. \quad (12)$$

Now, if we assume that $\bar{\varphi}_l(\tau, r, \theta, \phi) = \bar{\Phi}_l(\tau, r) \bar{G}_{l,m}(\theta, \phi)$, then the equation for $\bar{\Phi}_l(r, \tau)$ on the hypersurface Σ can be written as

$$\begin{aligned} & \frac{\partial^2 \bar{\Phi}_l}{\partial \tau^2} - \frac{2}{\tau} \frac{\partial \bar{\Phi}_l}{\partial \tau} - \frac{2}{r} \frac{\partial \bar{\Phi}_l}{\partial r} - \frac{\partial^2 \bar{\Phi}_l}{\partial r^2} - \left[\frac{l(l+1)}{r^2} - M^2 a^2(\tau) \right] \bar{\Phi}_l \\ &= \left(1 - \frac{J}{F} \right) \frac{\partial^2 \bar{\Phi}_l}{\partial \tau^2} - \left[\frac{2}{\tau} + \frac{1}{\sqrt{FJ}} \frac{\partial}{\partial \tau} \left(\frac{J^3}{F} \right)^{1/2} \right] \frac{\partial \bar{\Phi}_l}{\partial \tau} \\ &- M^2 (J - 1) \bar{\Phi}_l + \frac{1}{2} \left(\frac{1}{F} \frac{\partial F}{\partial r} + \frac{1}{J} \frac{\partial J}{\partial r} \right) \frac{\partial \bar{\Phi}_l}{\partial r}. \end{aligned} \quad (13)$$

Next, using the fact that ϵ is a small parameter, we propose the following expansion for $\bar{\Phi}_l$ in orders of ϵ :

$$\bar{\Phi}_l(r, \tau) = \bar{\Phi}_l^{(0)} + \bar{\Phi}_l^{(1)} + \bar{\Phi}_l^{(2)} + \dots \quad (14)$$

If we expand the right hand side of the equation (13) as powers of $\epsilon \ll 1$ [13], we obtain

$$\begin{aligned} & - 8 \left(\frac{\epsilon \tau}{2r} \right) \left[\frac{\partial^2 \bar{\Phi}_l^{(0)}}{\partial \tau^2} - \frac{1}{\tau} \frac{\partial \bar{\Phi}_l^{(0)}}{\partial \tau} - \frac{M^2}{2H^2 \tau^2} \bar{\Phi}_l^{(0)} \right] \\ & - 30 \left(\frac{\epsilon \tau}{2r} \right)^2 \left[\frac{\partial^2 \bar{\Phi}_l^{(1)}}{\partial \tau^2} - \frac{1}{15} \frac{1}{\tau} \frac{\partial \bar{\Phi}_l^{(1)}}{\partial \tau} - \frac{M^2}{5H^2 \tau^2} \bar{\Phi}_l^{(1)} \right] + \dots \end{aligned} \quad (15)$$

Thus, the spectrum (12) can be written using the expansion (14) as

$$\begin{aligned} \mathcal{P}_k(\tau) &= \sum_l \frac{(2l+1)}{4\pi} k^5 j_l^2(kr) \left[\bar{\Phi}_{kl}^{(0)} + \bar{\Phi}_{kl}^{(1)} + \dots \right] \left[\left(\bar{\Phi}_{kl}^{(0)} \right)^* + \left(\bar{\Phi}_{kl}^{(1)} \right)^* + \dots \right] \\ &= \frac{k^3}{2\pi^2} \left| \bar{\Phi}_{kl=0}^{(0)} \right|^2 + \frac{H^2}{4\pi^2} \epsilon \sum_{l=1}^\infty (2l+1) j_l^2(kr) \Delta_{kl}^{(1)} + \dots, \end{aligned} \quad (16)$$

where

$$\Delta_{kl}^{(1)} = \left(\frac{4\pi^2}{H^2\epsilon} \right) \frac{k^5}{4\pi} \left[\bar{\Phi}_{kl}^{(0)} \left(\bar{\Phi}_{kl}^{(1)} \right)^* + \bar{\Phi}_{kl}^{(1)} \left(\bar{\Phi}_{kl}^{(0)} \right)^* \right] = \frac{2\pi}{H^2\epsilon} k^5 \text{Re} \left[\bar{\Phi}_{kl}^{(1)} \left(\bar{\Phi}_{kl}^{(0)} \right)^* \right]. \quad (17)$$

Notice that the first term in (16) corresponds to $l = 0$, so that the zeroth order approximation in ϵ is due only to isotropic fluctuations. Terms with $l = 1$ correspond to dipoles and $l \geq 2$ are related to multipoles.

III. COARSE-GRAINING OF $\bar{\varphi}$

As it was shown in [11], the metric (7) written in the static coordinate chart (T, R) , describes an spherically symmetric object having properties of attractive and repulsive gravity, under the election of $\psi_0 = H^{-1}$. Specifically, at scales larger than the gravitational-antigravitational radius R_{ga} , gravity manifests itself as repulsive in nature. On the contrary, on scales smaller than R_{ga} gravity recovers its usual attractive behavior. In this section our goal is to study the evolution of the effective scalar field $\bar{\varphi}$ under the presence of such an object but in the dynamical coordinate chart (τ, \vec{r}) .

To study the evolution of the effective field $\bar{\varphi}(\tau, \vec{r})$ on scales larger than the gravitational-antigravitational radius r_{ga} we introduce the field

$$\bar{\varphi}_L(\tau, \vec{r}) = \int_{k_H}^{k_{Sch}} dk \sum_{l,m} \Theta_L(\sigma k_{ga} - k) \left[a_{klm} \bar{\Phi}_{klm}(\tau, \vec{r}) + a_{klm}^\dagger \bar{\Phi}_{klm}^*(\tau, \vec{r}) \right], \quad (18)$$

where Θ_L is denoting the heaviside function, and the wave number associated to the Hubble horizon is

$$k_H(\tau) \simeq 2\pi/[a(\tau)r_H] = -(2\pi)H\tau/r_H. \quad (19)$$

Furthermore, the time dependent wavenumber

$$k_{ga}(\tau) = [2\pi/(a(\tau)r_{ga})][(2a(\tau)r_{ga})/(2a(\tau)r_{ga} + Gm)]^2, \quad (20)$$

is the wave number associated to the gravitational-antigravitational radius r_{ga} , and σ is a dimensionless parameter that during inflation ranges in the interval $10^{-3} - 10^{-2}$.

Similarly, the evolution of the effective scalar field $\bar{\varphi}(\tau, \vec{r})$ on small scales: scales between the Schwarzschild radius r_{Sch} and the gravitational-antigravitational radius r_{ga} , can be described by the field

$$\bar{\varphi}_S(\tau, \vec{r}) = \int_{k_H}^{k_{Sch}} dk \sum_{l,m} \Theta_S(k - \sigma k_{ga}) \left[a_{klm} \bar{\Phi}_{klm}(\tau, \vec{r}) + a_{klm}^\dagger \bar{\Phi}_{klm}^*(\tau, \vec{r}) \right], \quad (21)$$

where Θ_S denotes the heaviside function and $k_{Sch} \simeq 8\pi a(\tau)r_{Sch}/(Gm)^2 = -8\pi r_{Sch}/[H\tau(Gm)^2]$ is the wave number associated to the Schwarzschild radius r_{Sch} . From the expressions (18) and (21) it can be easily seen that $\bar{\varphi}(\tau, \vec{r}) = \bar{\varphi}_L(\tau, \vec{r}) + \bar{\varphi}_S(\tau, \vec{r})$.

IV. SCALAR FIELD FLUCTUATIONS AT ZEROth ORDER IN ϵ

At zeroth order in the expansion (14), the equation (13) reduces to

$$\frac{\partial^2 \bar{\Phi}_l^{(0)}}{\partial \tau^2} - \frac{2}{\tau} \frac{\partial \bar{\Phi}_l^{(0)}}{\partial \tau} - \frac{2}{r} \frac{\partial \bar{\Phi}_l^{(0)}}{\partial r} - \frac{\partial^2 \bar{\Phi}_l^{(0)}}{\partial r^2} - \left[\frac{l(l+1)}{r^2} - M^2 a^2(\tau) \right] \bar{\Phi}_l^{(0)} = 0, \quad (22)$$

where for the zeroth approximation we must restrict to $l = 0$. Now in order to simplify the structure of (22), let us to introduce the field $\chi_{l=0}^{(0)}(\tau, r)$, with $\bar{\Phi}_{l=0}^{(0)}(\tau, r) = \tau \chi_{l=0}^{(0)}(\tau, r)$, so that the equation (22) can be written in the form

$$\frac{\partial^2 \chi_{l=0}^{(0)}}{\partial \tau^2} - \frac{2}{r} \frac{\partial \chi_{l=0}^{(0)}}{\partial r} - \frac{\partial^2 \chi_{l=0}^{(0)}}{\partial r^2} - m_{eff}^2(\tau) \chi_{l=0}^{(0)} = 0, \quad (23)$$

where $m_{eff}^2(\tau) = 2/\tau^2 - M^2/(H^2\tau^2)$ is the effective mass of the inflaton field. By means of the Bessel transformation

$$\chi_{l=0}^{(0)}(\tau, r) = \int_0^\infty dk k^2 j_{l=0}(kr) \xi_{kl=0}^{(0)}(\tau) \quad (24)$$

we derive from (23) the next equation for the modes ξ_{k0} :

$$\frac{\partial^2 \xi_{k0}^{(0)}}{\partial \tau^2} + [k^2 - m_{eff}^2(\tau)] \xi_{k0}^{(0)} = 0, \quad (25)$$

such that the modes of $\bar{\Phi}_{l=0}^{(0)}$ are given by $\bar{\Phi}_{k0}^{(0)} = \tau \xi_{k0}^{(0)}$. Thus solving (25) the normalized solution for the modes $\bar{\Phi}_{k0}^{(0)}$ has the form

$$\bar{\Phi}_{k0}^{(0)}(\tau) = A_1 (-\tau)^{3/2} \mathcal{H}_\nu^{(1)}[-k\tau] + A_2 (-\tau)^{3/2} \mathcal{H}_\nu^{(2)}[-k\tau], \quad (26)$$

Here, $\mathcal{H}_\nu^{(1,2)}[-k\tau]$ are respectively the first and second kind Hankel functions, $\nu^2 = \frac{9}{4} - \frac{M^2}{H^2}$, and the normalization constants are given by

$$A_2 = -\frac{\sqrt{\pi}H}{2} e^{-i\nu\pi/2}, \quad A_1 = 0. \quad (27)$$

Now we introduce the fields

$$\left[\chi_L^{(0)}\right]_{l=0}(\tau, r) = \int_{k_H}^{k_{Sch}} dk \Theta_L(\sigma k_{ga} - k) \left[a_{k0} j_0(kr) \xi_{k0}^{(0)}(\tau) + a_{k0}^\dagger j_0^*(kr) \xi_{k0}^{(0)*}(\tau) \right], \quad (28)$$

$$\left[\chi_S^{(0)}\right]_{l=0}(\tau, r) = \int_{k_H}^{k_{Sch}} dk \Theta_S(k - \sigma k_{ga}) \left[a_{k0} j_0(kr) \xi_{k0}^{(0)}(\tau) + a_{k0}^\dagger j_0^*(kr) \xi_{k0}^{(0)*}(\tau) \right], \quad (29)$$

where $\chi_{l=0}^{(0)}(\tau, r) = \left[\chi_L^{(0)}\right]_{l=0}(\tau, r) + \left[\chi_S^{(0)}\right]_{l=0}(\tau, r)$ and $\xi_{k0}^{(0)}(\tau) = \tau^{-1} \bar{\Phi}_{k0}^{(0)}(\tau)$. The equation of motion for $\left[\chi_L^{(0)}\right]_{l=0}$ is given by

$$\left[\ddot{\chi}_L^{(0)}\right]_{l=0} - m_{eff}^2(\tau) \left[\chi_L^{(0)}\right]_{l=0} = \sigma \ddot{k}_{ga} \eta_{l=0}^{(0)}(\tau, r) + \sigma \dot{k}_{ga} \lambda_{l=0}^{(0)}(\tau, r) + 2\sigma \dot{k}_{ga} \gamma_{l=0}^{(0)}(\tau, r), \quad (30)$$

where the stochastic operator fields $\eta_{l=0}^{(0)}$, $\lambda_{l=0}^{(0)}$ and $\gamma_{l=0}^{(0)}$ are defined as

$$\eta_{l=0}^{(0)}(\tau, r) = \int_{k_H}^{k_{Sch}} dk \delta(k - \sigma k_{ga}) \left[a_{k0} j_0(kr) \xi_{k0}^{(0)}(\tau) + a_{k0}^\dagger j_0^*(kr) \xi_{k0}^{(0)*}(\tau) \right], \quad (31)$$

$$\lambda_{l=0}^{(0)}(\tau, r) = \int_{k_H}^{k_{Sch}} dk \dot{\delta}(k - \sigma k_{ga}) \left[a_{k0} j_0(kr) \xi_{k0}^{(0)}(\tau) + a_{k0}^\dagger j_0^*(kr) \xi_{kl}^{(0)*}(\tau) \right], \quad (32)$$

$$\gamma_{l=0}^{(0)}(\tau, r) = \int_{k_H}^{k_{Sch}} dk \delta(k - \sigma k_{ga}) \left[a_{k0} j_0(kr) \dot{\xi}_{k0}^{(0)}(\tau) + a_{k0}^\dagger j_0^*(kr) \dot{\xi}_{k0}^{(0)*}(\tau) \right], \quad (33)$$

with the dot denoting $\partial/\partial\tau$. The field equation (30) can be expressed in the form

$$\left[\ddot{\chi}_L^{(0)}\right]_{l=0} - m_{eff}^2(\tau) \left[\chi_L^{(0)}\right]_{l=0} = \sigma \left[\frac{d}{d\tau} (\dot{k}_{ga} \eta_{l=0}^{(0)}) + 2\dot{k}_{ga} \gamma_{l=0}^{(0)} \right]. \quad (34)$$

This is a Kramers-like stochastic equation, that with the help of the auxiliary field: $u_{l=0}^{(0)} = \left[\dot{\chi}_L^{(0)}\right]_{l=0} - \sigma \dot{k}_{ga} \eta_{l=0}^{(0)}$, can be written as the first order stochastic system

$$\dot{u}_{l=0}^{(0)} = m_{eff}^2 \left[\chi_L^{(0)}\right]_{l=0} + 2\sigma \dot{k}_{ga} \gamma_{l=0}^{(0)}, \quad (35)$$

$$\left[\dot{\chi}_L^{(0)}\right]_{l=0} = u_{l=0}^{(0)} + \sigma \dot{k}_{ga} \eta_{l=0}^{(0)}, \quad (36)$$

The role that the noise $\gamma_{l=0}^{(0)}$ plays in this system, can be minimized in the system (35) and (36) when the condition $\dot{k}_{ga}^2 \langle (\gamma_{l=0}^{(0)})^2 \rangle \ll \ddot{k}_{ga}^2 \langle (\eta_{l=0}^{(0)})^2 \rangle$ is valid. This condition can be expressed as

$$\left| \frac{\dot{\xi}_{k0}^{(0)} (\dot{\xi}_{kl}^{(0)})^*}{\xi_{k0}^{(0)} (\xi_{k0}^{(0)})^*} \right|_{k=\sigma k_{ga}} \ll \left(\frac{\ddot{k}_{ga}}{\dot{k}_{ga}} \right)^2, \quad (37)$$

which is valid on large scales i.e. scales where $k_{ga}(\tau) < k < k_H(\tau)$, for k_H and k_{ga} given respectively by (19) and (20)². If this is the case, the system (35) and (36) can be approxi-

² When the background is an exact de Sitter space-time and the field is free, this condition is analogous to one obtained already by Mijic[14] in a different approach.

mated by

$$\dot{u}_{l=0}^{(0)} = m_{eff}^2 \left[\chi_L^{(0)} \right]_{l=0}, \quad (38)$$

$$\left[\dot{\chi}_L^{(0)} \right]_{l=0} = u_{l=0}^{(0)} + \sigma \dot{k}_{ga} \eta_{l=0}^{(0)}. \quad (39)$$

This is an stochastic two-dimensional Langevin equation with a noise $\eta_{l=0}^{(0)}$ which is gaussian and white in nature, as it is indicated by the following expressions:

$$\left\langle \eta_{l=0}^{(0)} \right\rangle = 0, \quad (40)$$

$$\left\langle (\eta_{l=0}^{(0)})^2 \right\rangle = 4\pi\sigma \frac{k_{ga}^2}{\dot{k}_{ga}} j_0(kr) j_0^*(kr) \xi_{k0}^{(0)} \xi_{k0}^{(0)*} \Big|_{k=\sigma k_{ga}} \delta(\tau - \tau'). \quad (41)$$

The correlation functions of $\eta_{l=0}^{(0)}$ and $\gamma_{l=0}^{(0)}$ have the same structure, similar to the momenta of a Gaussian white noise. The dynamics of the probability transition $\mathcal{P}_{l=0}^{(0)} \left[[\chi_L^{(0)}]_{l=0}, [u^{(0)}]_{l=0} \mid [\chi_L^{(0)}]_{l=0}, u_{l=0}^{(0)} \right]$ from an initial configuration $([\chi_L^{(0)}]_{l=0}, [u^{(0)}]_{l=0})$ to a configuration $(\chi_L^{(0)}, u^{(0)})$, is given by the Fokker-Planck equation:

$$\frac{\partial \mathcal{P}_{l=0}^{(0)}}{\partial \tau} = -u^{(0)} \frac{\partial \mathcal{P}_{l=0}^{(0)}}{\partial [\chi_L^{(0)}]_{l=0}} - m_{eff}^2 \left[\chi_L^{(0)} \right]_{l=0} \frac{\partial \mathcal{P}_{l=0}^{(0)}}{\partial u_{l=0}^{(0)}} + \frac{1}{2} D_{11}^{(0)} \frac{\partial^2 \mathcal{P}_{l=0}^{(0)}}{\partial [\chi_L^{(0)}]_{l=0}^2}, \quad (42)$$

where $D_{11}^{(0)} = \int (\sigma \dot{k}_{ga})^2 \left\langle (\eta_{l=0}^{(0)})^2 \right\rangle d\tau$ is the diffusion coefficient related to $[\chi_L^{(0)}]_{l=0}$. By using (41) the diffusion coefficient $D_{11}^{(0)}$ becomes

$$D_{11}^{(0)} = 4\pi\sigma^3 \dot{k}_{ga} k_{ga}^2 j_0(kr) j_0^*(kr) \xi_{k0}^{(0)} \xi_{k0}^{(0)*} \Big|_{k=\sigma k_{ga}}. \quad (43)$$

Hence, the dynamics of $\left\langle \left([\chi_L^{(0)}]_{l=0} \right)^2 \right\rangle = \int d[\chi_L^{(0)}]_{l=0} du_{l=0}^{(0)} \left([\chi_L^{(0)}]_{l=0} \right)^2 \mathcal{P}_{l=0}^{(0)}$ is given by the equation

$$\frac{d}{d\tau} \left\langle \left([\chi_L^{(0)}]_{l=0} \right)^2 \right\rangle = \frac{1}{2} D_{11}^{(0)}(\tau). \quad (44)$$

Now, in order to return to the original zeroth order scalar field, let us to use the expression $\bar{\Phi}_{l=0}^{(0)}(\tau, r) = \tau \chi_{l=0}^{(0)}(\tau, r)$ in (44) to obtain

$$\frac{d}{d\tau} \left\langle \left([\bar{\phi}_L^{(0)}]_{l=0} \right)^2 \right\rangle = \frac{2}{\tau} \left\langle \left([\bar{\phi}_L^{(0)}]_{l=0} \right)^2 \right\rangle + \frac{1}{2} \tau^2 D_{11}^{(0)}(\tau). \quad (45)$$

The general solution of (45), is then

$$\left\langle \left([\bar{\phi}_L^{(0)}]_{l=0} \right)^2 \right\rangle = \frac{1}{2} \tau^2 \left[\int^\tau D_{11}^{(0)}(\tau') d\tau' + C \right], \quad (46)$$

with C an integration constant. Next, we employ the relation

$$\left\langle \left(\bar{\varphi}_L^{(0)} \right)^2 \right\rangle = \left(\frac{1}{4\pi} \right) \left\langle \left(\left[\bar{\Phi}_L^{(0)} \right]_{l=0} \right)^2 \right\rangle, \quad (47)$$

where we have used the addition theorem of the spherical harmonics, to derive the equation

$$\left\langle \left(\bar{\varphi}_L^{(0)} \right)^2 \right\rangle = \left(\frac{1}{8\pi} \right) \tau^2 \left[\int D_{11}^{(0)}(\tau) d\tau + C \right]. \quad (48)$$

This equation, give us in principle the squared fluctuations of $\bar{\varphi}_L$ on large scales. Employing (26) and (43), the expression (48) with $C = 0$, can be approximated on the IR sector as

$$\left\langle \left(\bar{\varphi}_L^{(0)} \right)^2 \right\rangle \Big|_{kr \ll 1} \simeq \left(\frac{H^2}{2\pi} \right) 2^{2(\nu-1)} \sigma^{3-2\nu} \Gamma^2(\nu) (-\tau)^{3-2\nu} \int \frac{dk_{ga}}{k_{ga}} k_{ga}^{3-2\nu} \quad (49)$$

where we have used the asymptotic expansion $j_0(kr)|_{kr \ll 1} \simeq 1$. The spectrum derived from (49) at zeroth order (i.e. for $l = 0$), has the form

$$\mathcal{P}_{k_{ga}}^{(0)}(\tau) \simeq 2^{2(\nu-1)} \Gamma^2(\nu) \left(\frac{H^2}{2\pi} \right) [\sigma(-\tau) k_{ga}]^{3-2\nu}, \quad (50)$$

which results scale invariant when $\nu = 3/2$ and when this is the case its time dependence disappears. This spectrum is similar to whole obtained in a previous work[15], but in a different manner. It is characteristic of an universe governed by a cosmological constant, in agreement with one expects for a de Sitter expansion of the universe. However, for spectrums close, but different to the Harrison-Zeldovich, the situation changes, because it becomes sensitive to the wavenumber k_{ga} and therefore with the mass of the BH. Furthermore, the amplitude of this spectrum tends to zero (as $\tau \rightarrow 0$), for $\nu < 3/2$.

V. FINAL COMMENTS

We have developed a stochastic approach to study scalar field fluctuations of the inflaton field in an early inflationary universe, which is described by an effective 4D SdS metric. The cosmological metric was obtained using planar coordinate transformations on a 5D Ricci-flat Schwarzschild-de Sitter (SdS) static metric (1), for a SdS BH. From the dynamical point of view, the effective 4D cosmological metric (7) describes the collapse of the universe on scales $k \gg k_{ga}$ and an accelerated expansion for scales much bigger than the gravitational - antigravitational radius r_{ga} , which is related with the wavenumber k_{ga} .

The main difference with earlier stochastic approaches to inflation where the window function is defined on the Hubble horizon is that, in our approach [see eq. (18)], the coarse-grained field is defined using a window function $\Theta_L(\sigma k_{ga} - k)$, which takes into account only modes with wavelengths larger than the gravitational - antigravitational radius r_{ga} . This fact indicates the scale for which the universe is starting to expand accelerated. On smaller scales the universe is collapsing due to the attraction of the BH. However, on larger scales gravitation is repulsive and drives inflation. For the limit case in which this mass is very small, $Gm/(2ar_{ga}) \ll 1$, we obtain that $k_{ga}|_{Gm/(2ar_{ga}) \ll 1} \rightarrow k_H$, and our result agrees completely with whole of the squared field fluctuations of a de Sitter expansion during the inflationary stage when the horizon entry. For $r \rightarrow \infty$ J and F approach to $a^2(\tau)$ and the metric (7) describes a de Sitter expansion. However, for very large (but finite) cosmological scales the spectrum is not exactly scale independent, because becomes sensitive to the wavenumber k_{ga} . For $\nu < 3/2$ the spectral index $n_s = 3 - 2\nu$ is positive and the amplitude decreases as $\tau \rightarrow 0$.

We have restricted our stochastic study to very small fluctuations on cosmological scales. A more profound study should necessarily include higher orders in the expansion (14) of $\bar{\phi}_l$ in the equation (13), which takes into account multipolar expansion due to non-gaussian noises.

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